THE *SCHRÖDINGER* NONLINEAR PARTIAL DIFFERENTIAL EQUATION SOLUTION IN QUANTUM PHYSIC BY NEW APPROACH AYM

AKBARI, M. R.[1*] – AKBARI, S.[2] – KALANTARI, E.[2]

[1] *Faculty of Chemical Engineering, University of Tehran, Tehran, Iran.*

[2] *Faculty of Chemical Engineering, Islamic Azad University, Ghaemshahr, Iran.*

**Corresponding author*
e-mail: Akbari_hamid46[at]yahoo.com

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Abstract. In this paper, we investigate and solve a complicated highly nonlinear differential equations of ***Schrödinger equation*** by analytical solving of new method which we named it ***AYM*** (Akbari Yasna's Method).The Schrödinger equation is the fundamental equation of physics for describing quantum mechanical behavior. It is also often called the ***Schrödinger*** wave equation, and is a partial differential equation that describes how the wave function of a physical system evolves over time. The *Schrödinger* equation, the fundamental equation of the science of submicroscopic phenomena known as quantum mechanics. And the Schrödinger equation gives exact solutions only for nuclei with one electron: H, He^+, $Li2^+$, $Be3^+$, $B4^+$, $C5^+$, etc.The equation, developed (1926) by the Austrian physicist Erwin Schrödinger, has the same central importance to quantum mechanics as Newton's laws of motion have for the large-scale phenomena of classical mechanics.
Keywords: *new method, Akbari-Yasna-Method (AYM), Schrödinger Equation, nonlinear partial differential equation, quantum mechanical*

Introduction

In the paper, our aims introduce of accuracy, capabilities and power for solving complicated nonlinear partial differential of Schrödinger equation in the Quantum mechanical by *AYM* method. The Schrödinger equation is a nonlinear partial differential equation that describes the wave function or state function of a quantum-mechanical system. It is a key result in quantum mechanics, and its discovery was a significant landmark in the development of the subject. The equation is named after Erwin Schrödinger, who postulated the equation in 1925, and published it in 1926, forming the basis for the work that resulted in his Nobel Prize in Physics in 1933. Solving by *AYM* method can be successfully applied in the various engineering fields such as mechanics (solid, fluid and waves), electronics, petroleum industry, and also in applied sciences (physics and quantum), including the Schrödinger partial differential equation (Briggs and Rost, 2001),economics and and etc. It is worth noting that these method is convergent at any form of nonlinear differential equations, including any number of initial and boundary conditions. During the solution procedure, it is not required to convert or simplify the exponential, trigonometric and logarithmic terms, which enables the user to obtain a highly precise solution. Besides, the methodology behind these techniques are completely understandable, easy to use, and users with common knowledge of mathematics will be capable of solving the most complicated equations at low calculation cost.As all experts know most of engineering actual systems behavior in practical are nonlinear process and analytical scrutiny these nonlinear problems are difficult or sometimes impossible. Our purpose is to enhance the ability of solving the mentioned nonlinear differential equations at chemical engineering and similar issues with a simple and innovative approach which entitled "Akbari-Kalantari-Leila Method" or "AKLM". He's Amplitude Frequency Formulation method (Zen et al., 2009; He, 2008; He, 1998) which was first presented by Ji-Huan He gives convergent successive approximations of the exact solution and Homotopy perturbation technique HPM (He, 1999). It is necessary to mention that the above methods do not have this ability to gain the solution of the presented problem in high precision and accuracy so nonlinear differential equations such as the presented problem in this case study should be solved by utilizing new approaches like AGM (Ahmadi et al., 2015; Akbari et al., 2020a; 2014a; 2014b; 2014c; 2014d; 2014e; Rostami et al., 2014) that created by Mohammadreza Akbari(in 2014).In recent years, analytical methods in solving nonlinear differential equations have been presented and created by Mohammadreza Akbari, these methods are called AYM (Akbari et al., 2020b) (Akbari Yasna's Method in April 2020) and ASM (Akbari et al., 2020c) (Akbari Sara's Method in August 2019) and AKLM (Akbari Kalantari Leila Method in August 2020).These example somehow can be considered as complicated cases to deal with for all of the existed analytical methods especially in the design reactor in chemical engineering, which means old methods cannot resolve them precisely or even solve them in a real domain.

Mathematical formulation of the problem

The fundamental the Schrödinger nonlinear differential equation in the quantum physics is as follows:

$$i\,h\frac{\partial\Psi}{\partial t}=\left[\frac{-h^2}{2m}\nabla^2+V(x,y,z,t)\right]\Psi \qquad \text{Eq. 1}$$

$$\Psi=\Psi(x,y,z,t)\ \ ,\ V(x,y,z,t)=k\ \Psi(x,y,z,t)^2 \qquad \text{Eq. 2}$$

And Laplacian operator as:

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2} \qquad \text{Eq. 3}$$

Here the parameters of the *Schrödinger* equation Eq.(1) are ***h*** is the reduced Planck constant; ***m*** is the electron mass, ∇ is the Laplacian operator; ***Ψ*** is the wave function; ***V*** is the potential energy; ***k*** is the potential energy nonlinearity coefficient; and ***i*** is the imaginary conmplex function.

Boundary and initial conditions the Schrödinger partial differential equation as follows;

$$\begin{aligned}&bc,ic:\ \Psi(0,y,z,t)=0,\ \Psi(a,y,z,t)=0,\ \Psi(x,0,z,t)=0\\&\Psi(x,b,z,t)=0,\ \Psi(x,y,0,t)=0,\ \Psi(x,y,c,t)=0\\&\Psi(x,y,z,0)=uo\end{aligned} \qquad \text{Eq. 4}$$

Value *a*, *b*, *c* are distance of boundary conditions, and uo is potential energy initial value. Output of the solution process by new approach AYM (Akbari Yasna's Method) for the Schrödinger partial nonlinear differential equation Eq.(1), according to the boundary conditions Eq.(4) is achieved as follows;

$$\Psi(x,y,z,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty} f(r)\ \times$$
$$e^{\frac{i}{2a2\ b2\ c2\ m}\left\{-\xi t+2\, m\, i\, a2\ b2\ c2\ \mathrm{LambertW}\left(-\frac{i t}{16\lambda}\ C\pi^3\, h\, k\, n\, p\, e^{\frac{-i\xi t}{2\, a2\ b2\ c2\ m}}\right)\right\}}$$

Eq. 5

And parameters and shape functions Eq(1) according to boundary conditions Eqs.(4) as follows;

$$f(r)=C\sin\left(\frac{n\pi x}{a}\right)\sin\left(\frac{m\pi y}{b}\right)\sin\left(\frac{p\pi z}{c}\right);\ C=\frac{-8\lambda}{p\, m\, n\, \pi^3}$$
$$\xi=\pi^2\, h\left(a^2 b^2 p^2+a^2 c^2 m^2+b^2 c^2 n^2\right)$$

Eq. 6

$$\lambda=-1+(-1)^{m+n+p}-(-1)^{n+p}-(-1)^{m+p}$$
$$-(-1)^{m+n}+(-1)^{p}+(-1)^{n}+(-1)^{m}$$

Eq. 7

Values n,m,p are the wave function produces Quantum numbers ***n***, ***m***, ***p*** = 1,2,3…
And so the LambertW function as follows:

$$\mathrm{LambertW}(t)=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}n^{n-2}}{(n-1)!}t^n=t-t^2+\frac{3}{2}t^3-\frac{8}{3}t^4+\frac{125}{24}t^5\ldots$$

Eq. 8

We assume the physical values are with metric units as follows;

$$a := 1;\ b := 2;\ c := 1;\ k := -100000000;\ m := 0.00000000000001$$
$$h := 1.105457\cdot 10^{-34};\ uo := 10000000000;\ t0 := 5000,\ n := 1;\ p := 1;\ q := 1$$

Eq. 9

According to the physical values Eq.(9) the solution of the Schrödinger partial differential equation Eq.(1), wave function at (*x*=0.5,*y*=0.6, *z*=0.5) is obtained as follows (*Figure 1*);

$$\Psi(t)=-\ 0.0000764\ i\ \times$$
$$e^{-1.25\, 10^{13}\, i\left[8.72834\, 10^{-33}\, t-8.\, 10^{-14}\, i\, \mathrm{LambertW}\left\{(-0.0055273\, i)\, t\, e^{-1.091\, 10^{-19}\, i\, t}\right\}\right]}$$

Eq. 10

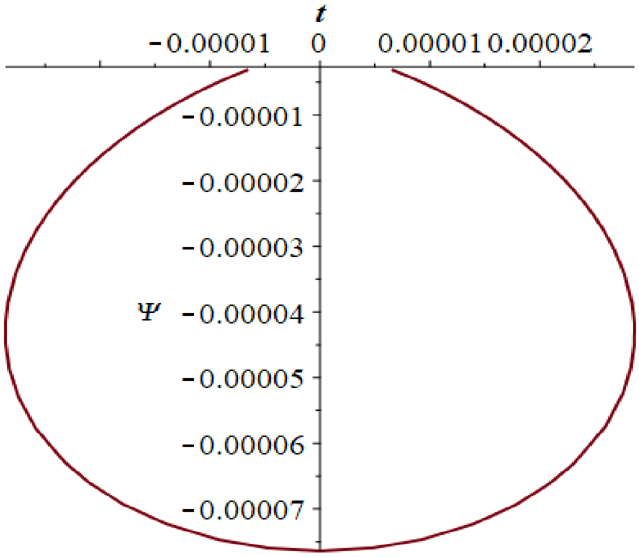

Figure 1. *Wave function by AYM method of Eq. 10.*

And the energy eigenvalue at (*x*=0.5,*y*=0.6, *z*=0.5) as follows;

$$E := i\,h\,\frac{\partial \Psi(x, y, z, t)}{\partial t} \qquad \text{Eq. 11}$$

According to the Eq.(10) and Eq(11), we have (*Figure 2*):

$$\begin{aligned} E = {} & -9.21392810^{-58}\, i\, e^{-1.09104233\, 10^{-19}\, i\, t} \\ & e^{-\mathrm{LambertW}\left(-0.0055272851\, t\, e^{-1.091\, 10^{-19}\, i\, t}\right)} \\ & - \left(8.44507\, 10^{-39}\, e^{-1.091\, 10^{-19}\, i\, t}\, e^{-\mathrm{LambertW}\left(-0.0055273\, i\, t\, e^{-1.091\, 10^{-1}}\right.}\right. \\ & -0.0055273\, i\, t\, e^{-1.09104233\, 10^{-19}\, i\, t}\Big)\Big)\, t\Big) \\ & + \left(9.21393\, 10^{-58}\, i\, e^{-1.09110^{-19}\, i\, t}\, e^{-\mathrm{LambertW}\left(-0.0055273\, i\, t\, e^{-1.09104233}\right.}\right. \\ & -0.00553\, i\, t\, e^{-1.091\, 10^{-19}\, i\, t}\Big)\Big) \end{aligned} \qquad \text{Eq. 12}$$

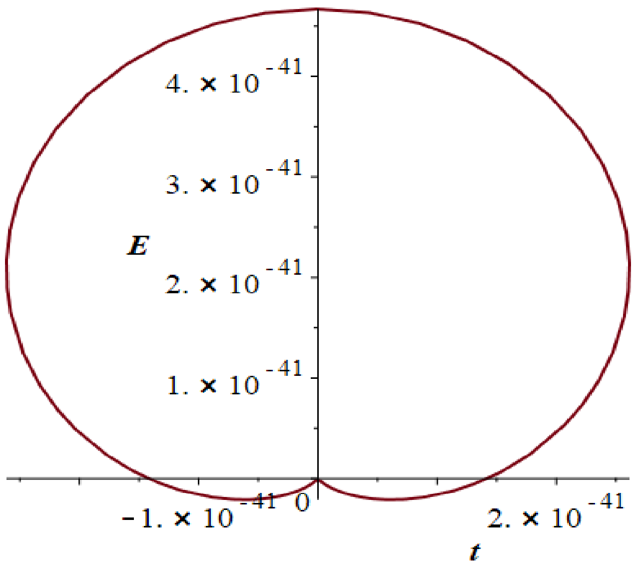

Figure 2. *Energy eigenvalue by AYM method of Eq. 12.*

Energy diagrams at different intervals of boundary conditions (***x*=*a*, *y*=*b*, *z*=*c***) of the equation obtained by the AYM method as follows (*Figure 3*):

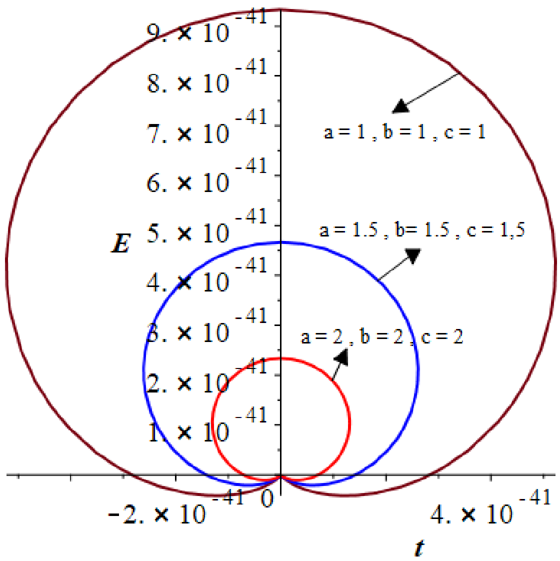

Figure 3. *Energies eigenvalue by AYM method for different value of boundary conditions.*

The graph of the real value of the energy function of Eq.(12) for values (*x*=0.5,*y*=0.6, *z*=0.5) is as follows (*Figure 4*);

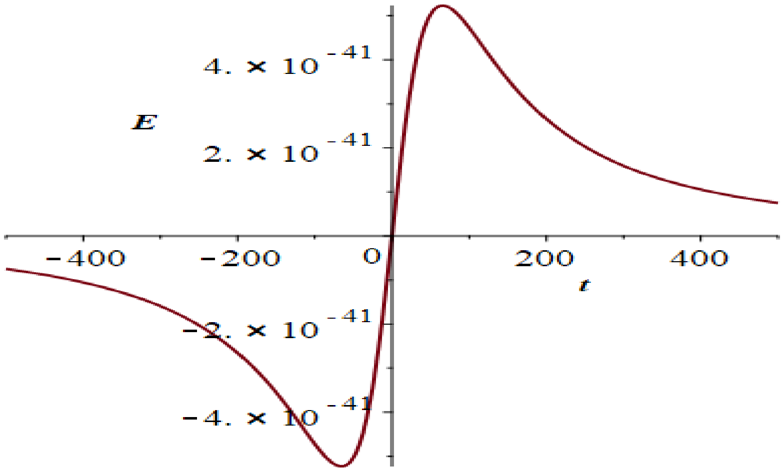

Figure 4. *Real value of the energy function of Eq. 12 by AYM method.*

The graph of the real value of the energy function of Eq.(12) for values (t =10sec,y=0.6, z=0.5) in the direction of (***x***) is as follows (*Figure 5*):

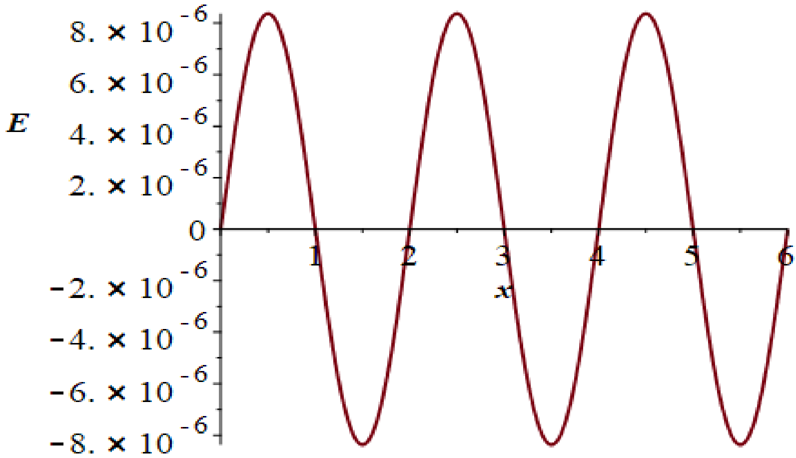

Figure 5. *Real value of the energy function of Eq. 12 by AYM method.*

Conclusion

In this article,we proved that with this new method,also we were able to simply solve the *Schrödinger* partial differential equation analytically. AYM method can all kinds of complicated nonlinear partial differential equations easily solve analytically. Obviously, most of the physical phenomena are nonlinear, so it is quite difficult to study and analyze nonlinear mathematical equations in this area, also we wanted to demonstrate the strength, capability and flexibility of the new *AYM* method(Akbari-Yasna's Method). This method is newly created and it can have high power in analytical solution of all kinds of industrial and practical problems in engineering fields and basic sciences for complex nonlinear differential equations.

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Conflict of Interest

The author confirm there are no conflict of interest with any parties involve in this research.

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