HYBRID QUANTUM ALGORITHMS FOR AUTONOMOUS VEHICLE NAVIGATION: DIRECTED PATH PLANNING WITH GROVER-QAOA

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Abstract. Path planning in complex and obstacle-constrained environments remains a challenging combinatorial optimization problem due to the exponential growth of the solution space and the limitations of classical algorithms in dynamic decision-making scenarios. Traditional approaches such as Dijkstra or A* may experience high computational costs and susceptibility to local optima when dealing with large-scale directed graphs containing obstacles. This study proposes a hybrid quantum computing approach that integrates the Grover Search Algorithm and the Quantum Approximate Optimization Algorithm (QAOA) to address optimal path planning on obstacle-oriented directed graphs. The Grover algorithm is first employed as an oracle-based filtering mechanism to detect and eliminate obstacle-containing edges, thereby generating a valid subgraph of feasible paths. Following this preprocessing step, QAOA is applied to the filtered graph within a nonlinear optimization framework to determine the minimum-cost path between source and destination nodes. The optimization model represents the graph as G=(V,E), where binary decision variables indicate whether an edge is included in the selected route while satisfying flow conservation constraints. By narrowing the solution space through Grover-based obstacle detection and performing optimization only on valid edges, the proposed method significantly reduces computational complexity compared to conventional approaches. Experimental analysis demonstrates that the hybrid quantum model effectively identifies optimal routes and adapts to different obstacle configurations while maintaining efficient convergence behavior. The results highlight the potential of integrating quantum search and quantum optimization techniques for solving combinatorial graph problems. This work contributes to the growing body of research on quantum-assisted optimization and demonstrates the feasibility of applying hybrid quantum algorithms to directed path planning problems relevant to autonomous navigation systems.
Keywords: *quantum computing, automated navigation, quantum optimization, algorithms*

Introduction

Combinatorial optimization problems are central to decision-making processes in complex systems and often appear as structures where the solution space grows exponentially. In particular, path planning problems defined on directed and constrained graph structures create a fundamental need for solutions in many fields, from logistics to robotics, communication networks to mobile autonomous systems (Arora and Barak, 2009; Papadimitriou and Steiglitz, 1998). These types of problems fall into the NP-hard category under classical computational paradigms, and the size of the solution space directly limits the performance of algorithms (Gungor and Karakose, 2025; Korte and Vygen, 2008). While classical search and optimization algorithms are effective in structured or deterministic environments, they can be insufficient in environments with factors such as environmental uncertainty, topological obstacles, and temporal variability due to constraints such as the need for recalculation, the risk of getting stuck in a local optimum, and high spatiotemporal complexity (Cormen et al., 2022). These

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challenges can make it impossible to generate solutions, especially in dynamic or obstacle-filled environments.

In recent years, advances in quantum computing have paved the way for the development of alternative solutions to such challenges. The applicability of quantum-inspired optimization algorithms to autonomous vehicle navigation and guided path planning problems has been validated in the literature (Yetis and Karakose, 2021). To reduce high programming costs in path planning rates, quantum-classical hybrid frameworks offer a solution for autonomous systems (Bar et al., 2024). Hybrid algorithms, particularly those specific to the Noisy Intermediate-Scale Quantum (NISQ) era, combine the advantages of both classical and quantum approaches, offering promising results for high-dimensional search and optimization problems (Bharti et al., 2022; Preskill, 2018). Two prominent quantum algorithms in this context, Grover Search Algorithm and Quantum Approximate Optimization Algorithm (QAOA), are used complementarily to eliminate invalid solutions and optimize valid solutions, respectively (Brassard et al., 2000; Grover, 1996).

The Grover algorithm offers a highly efficient solution for filtering constrained spaces such as obstacle detection, thanks to its quadratic acceleration O ($\sqrt{N}$) compared to classical linear searches (Fitzek et al., 2024). In particular, by integrating it with quantum oracle functions, it becomes possible to systematically exclude invalid edges via superposition. In the literature, the Grover algorithm has been successfully applied in many areas, from database searching to classification, from constrained logic problems to combinatorial selection structures (Boyer et al., 1998). The QAOA algorithm has a hybrid structure where parametric quantum circuits are trained with classical optimization and uses both the problem Hamiltonian and the driver Hamiltonian (Farhi et al., 2014). The success of QAOA has been repeatedly verified in combinatorial graph problems such as Max-Cut, Vertex Cover, independent sets, and job assignment (Zhou et al., 2020; Wang et al. 2018). However, in graphical structures involving directed and constrained paths, direct application of QAOA can lead to the evaluation of invalid solutions. Quantum-based solutions to such problems are supported by hybrid models that involve filtering out invalid solutions with the Grover search algorithm and optimizing valid solutions with QAOA. The literature includes studies that visually present the parametric circuit structure of QAOA and its interaction with classical optimization. The following figure shows a QAOA flowchart that exemplifies this architecture (*Figure 1*). This study proposes a two-stage hybrid quantum approach to solve the shortest path problem between starting and destination nodes on a directed graph constrained by obstacles. This approach involves filtering obstacle edges using the Grover algorithm and then optimizing only the valid subgraph using QAOA. The proposed model both narrows the solution space via Grover and increases efficiency by limiting the parameter training process of QAOA to only physically feasible paths. It has been observed that such Grover-QAOA hybrid algorithms have been applied to path planning problems in directed graph structures containing obstacles in a limited number of studies (Wang et al., 2018).

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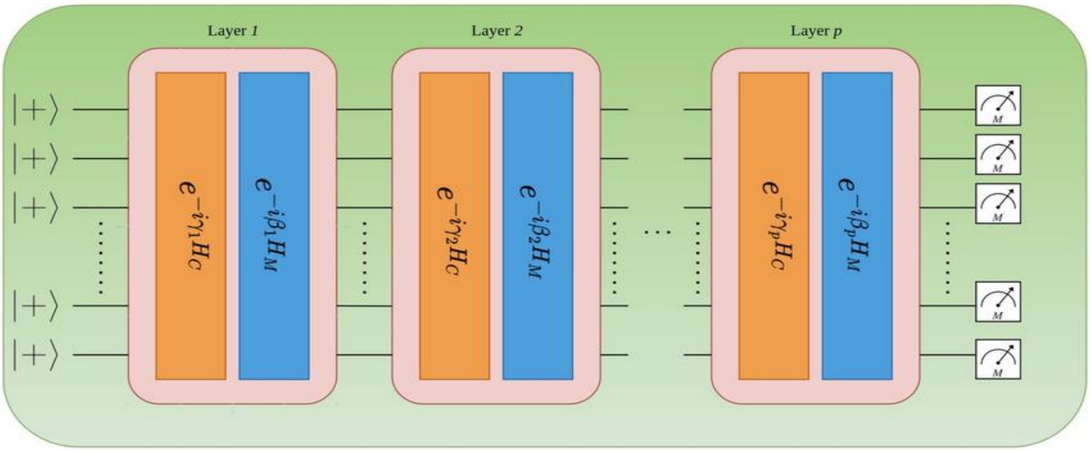

Figure 1. *Interaction between the circuit structure of QAOA and the classical optimization process.*
Source: Pellow-Jarman et al. (2024).

Materials and Methods

This study utilizes hybrid quantum algorithms to solve the shortest path problem on a directed and obstructed graph. Considering the computational complexity and time requirements of classical computational methods, Grover's quantum search algorithm and the Approximate Quantum Optimization Algorithm (QAOA)) are integrated. This approach enables both filtering of obstructed regions and determination of the optimal path on high-dimensional and complex graphs using quantum acceleration. The graph representation is in the form G = (V, E) , where V represents the set of nodes and E represents the set of edges. Edges determine the transitions between nodes, and (i,j)∈E a binary decision variable $x_{ij} \in \{0,1\}$ is defined for each edge, where $x_{ij} \in 1$ indicates that it is part of the chosen path. The objective function of the optimization is formulated to minimize the total cost of the chosen path.

$$\min \sum_{(i,j)\in E} C_{ij} \cdot x_{ij} \qquad \text{Eq. (1)}$$

Where, C_{ij}, represents the transition cost on the edge (i,j) . For this purpose, flow conservation constraints have been applied. Only one edge should exit the starting node, only one edge should enter the destination node, and the sum of entry and exit edges should be equal at intermediate nodes (Wang et al., 2018). These constraints are expressed by the following linear equations (Eq. (2):

$$\sum_{j:(i,j)\in E} x_{ij} - \sum_{k:(k,i)\in E} x_{ki} = f(x) = \begin{cases} 1, & i = s \\ -1 & i = t \\ 0, & i \in V\backslash\{s,t\} \end{cases} \qquad \text{Eq. (2)}$$

Where, s and t represent the source and destination nodes, respectively. The Grover algorithm is used for the detection and exclusion of obstacle edges. The Grover algorithm implements the "does it contain obstacles?" function, expressed as a truth table, as a quantum oracle and identifies invalid edges with $O(\sqrt{N})$ complexity. This

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provides a significant speedup compared to classical linear search (Montanaro, 2016). The oracle function allows the comparison of edges with the obstacle database, and control operations are performed on the quantum bit index (qubit register) to filter out obstacle edges. Thus, regions where passage is not possible on the map are quickly eliminated. After pre-filtering is completed with the Grover algorithm, QAOA is applied only to the subgraph formed by the valid edges. QAOA enables the determination of the lowest-cost path by filtering possible paths in the solution space through parametric quantum circuits (Wang et al., 2018). The basic mechanism of QAOA is the transformations between the problem Hamiltonian (Hc) and the driving Hamiltonian ( ) H_B). The problem Hamiltonian encodes the objective function as a quantum mechanical energy level (Eq. (3):

$$Hc = \sum_{(i,j)\in E} C_{ij} \frac{1 - z_i z_J}{2} \qquad \text{Eq. (3)}$$

Here z_i, and z_J are Pauli-Z operators, and C_{ij} represents the edge cost. The driver Hamiltonian, which enables exploration in the solution space of the quantum system, is defined in the following form (Eq. (4):

$$H_B = \sum_{i} x_i \qquad \text{Eq. (4)}$$

Here x_i, is the Pauli-X operator and is used to filter the qubits. The parameterized quantum circuit is constructed by successively applying these Hamiltonians, and the parameters are updated using classical optimization methods. Thanks to this hybrid loop, the algorithm iteratively progresses until it finds the lowest energy state (optimum solution) (Zhou et al., 2020). The proposed method utilizes the unique acceleration potential of quantum algorithms to produce efficient solutions to large-scale and complex obstacle graph problems that classical optimization approaches struggle to overcome (*Figure 2*). Recent studies have also reported that QAOA has achieved significant success in route planning, task assignment, and robotic navigation problems (Fitzek et al., 2024). Furthermore, reducing the solution space by pre-filtering with the Grover algorithm stands out as an innovative approach that increases the algorithm's efficiency and accuracy.

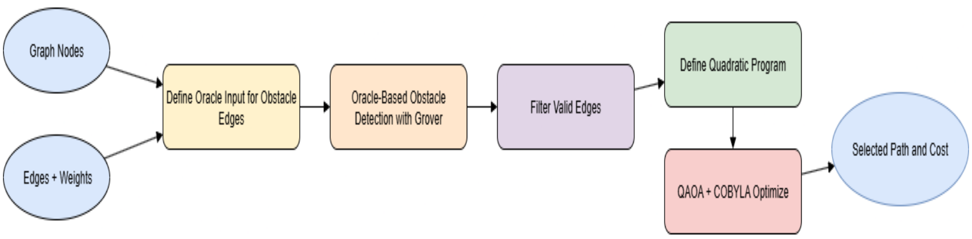

Figure 2. *Flowchart of the method.*

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Results and Discussion

This study demonstrates the advantages of quantum algorithms, particularly in combinatorial optimization problems such as path planning and obstacle detection. Computational costs and the risk of getting stuck in local minima, which traditional algorithms encounter especially when the problem space grows, can be significantly reduced with quantum algorithms. In this context, the QAOA (Quantum Approximate Optimization Algorithm) applied both minimized the targeted route cost through Hamiltonian modeling and generated optimum solutions based on valid path sets (Farhi et al., 2014) (*Figure 3*).

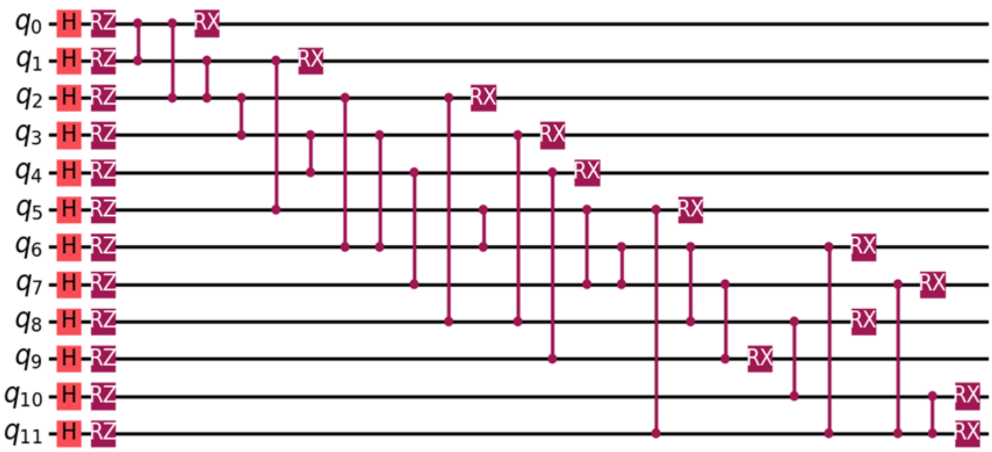

Figure 3. *Simplified QAOA circuit.*

The circuit presented in this study is a non-parametric and simplified QAOA circuit modeled on a 12-qubit system. This developed architecture is designed to offer a solution approach specifically for path planning problems, particularly for autonomous vehicles. The circuit structure aims to reduce the computational complexity encountered in classical path planning algorithms and to provide a more efficient solution space exploration by leveraging quantum supremacy. It begins by placing the system into a superposition state using Hadamard gates applied to each qubit. Subsequently, non-parametric RZ transformations applied to each qubit universally encode the problem with phase information. Then, numerous controlled-Z (CZ) gates, representing the connections and constraints on the path, interact between specific qubits in pairs to construct the problem Hamiltonian. This structure models the crossings, obstacles, and potential collisions on the path. RX gates, placed on specific qubits, represent the mixer Hamiltonian and enable the system to explore a wider solution space within quantum superpositions. The circuit was simplified by omitting the classical optimization step and redesigned to visually represent only the gate structures. The resulting circuit architecture provides a visual and structural example of how QAOA can be implemented in path planning problems for autonomous vehicles. This design sheds light on both the theoretical analysis of the algorithm and its applicability in physical quantum systems. The image below shows the optimal solution path obtained in a graph-based path planning problem where all edges are considered valid without defining any obstacles. This scenario represents the algorithm's baseline scenario. During the solution process, the accessibility of all edges allowed the algorithm to operate on a fully connected solution space. Thus, the resulting path was determined to be the route with the lowest total cost (*Figure 4, Figure 5* and *Figure 6*).

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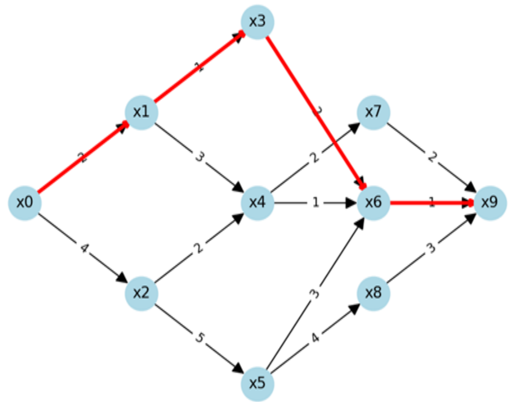

Figure 4. *Path planning without obstacles.*

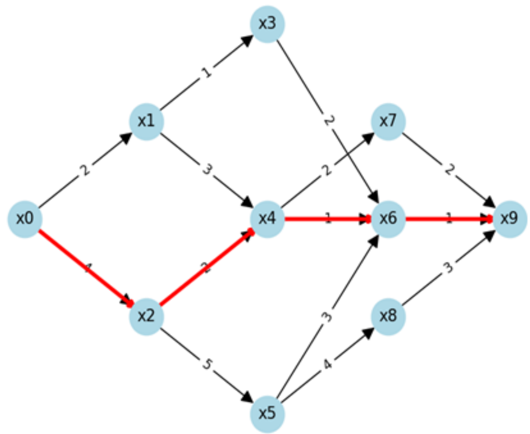

Figure 5. *Alternative Path Solutions via Constraint Integration 1.*

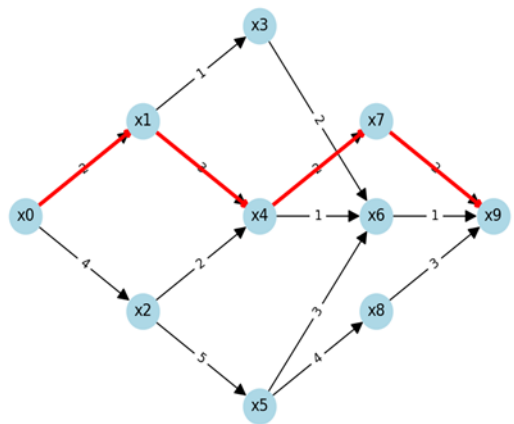

Figure 6. *Alternative Path Solutions via Constraint Integration 2.*

The path shown in bold red in the image represents the optimal route reached by the system by optimizing its minimum cost function. This structure forms the basis for analyzing how the algorithm develops different path strategies when it gains obstacle awareness, by comparing it with obstacle scenarios that will be defined later. Furthermore, this solution can be used as a reference point for comparatively examining

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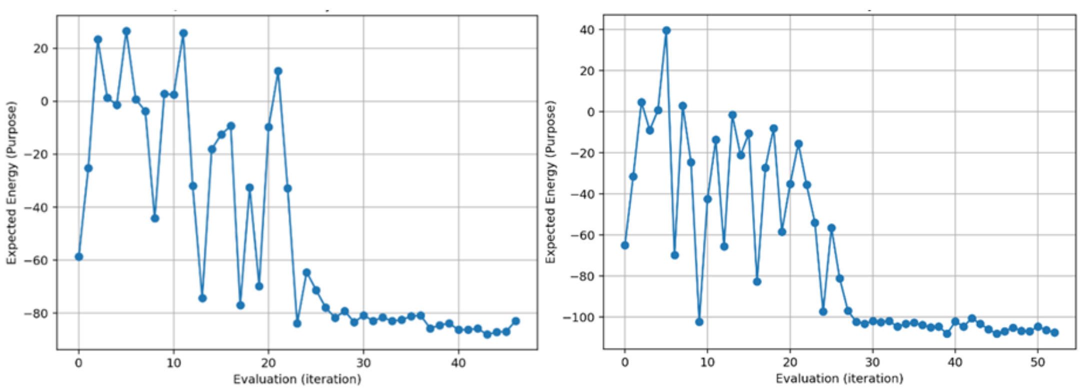
how the integration of quantum algorithms with hybrid architectures provides flexibility and contextual adaptation compared to classical solutions. The path shown in bold red in the image represents the optimal route reached by the system by optimizing its minimum cost function. This structure forms the basis for analyzing how the algorithm develops different path strategies when it gains obstacle awareness, by comparing it with obstacle scenarios that will be defined later. Furthermore, this solution can be used as a reference point for comparatively examining how the integration of quantum algorithms with hybrid architectures provides flexibility and contextual adaptation compared to classical solutions.

In both visuals, the solutions are highlighted with thick red edges on the graph, clearly demonstrating the algorithm's obstacle-aware path planning capability. In the first scenario, edges (x0, x1) and (x8, x9) are identified as obstacles. In the second scenario, edge sets (x1, x3) and (x4, x6) are excluded. This structure demonstrates the system's reconfigurability under different conditions and the applicability of quantum algorithms combined with the agility of the Grover-assisted discrete optimization approach. The QAOA + COBYLA energy convergence curves obtained under different obstacle configurations reveal that obstacle density has a significant effect on the dynamics of the hybrid quantum-classical optimization process. The parametric quantum circuit of QAOA was constructed by transforming the problem Hamiltonian into the Ising form, which includes both the cost function and the constraints via penalty terms. It was classically tuned by the COBYLA optimizer. In the first scenario, with obstructed edges-(x0, x1) and (x8, x9)-the expected energy values obtained from the quantum circuit started with high-amplitude fluctuations and rapidly converged to a minimum around -85 from approximately the 25th iteration. In the second scenario, with obstructed edges –(x1, x3) and (x4, x6) – the initial energy variance varied more widely, the minimum energy value was obtained at approximately -105, and the convergence time was extended to ~30 iterations. This difference, the change in forbidden edges, makes the energy landscape of the problem Hamiltonian more rugged, forcing the quantum circuit to explore the solution superpositions it generates over a wider parameter space. This delays the convergence of the optimization process, causing it to terminate at a different minimum energy level (*Figure 7*).



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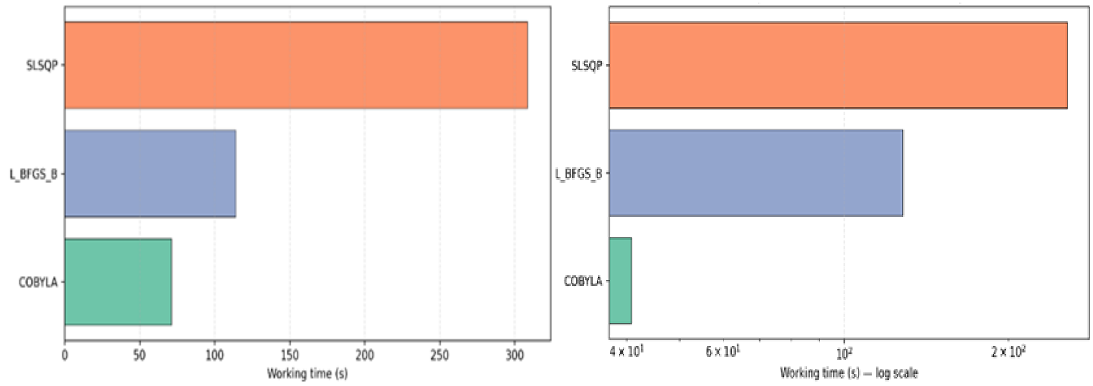

Figure 7. *QAOA optimizer comparison.*

The two graphs above illustrate the effect of optimizer selection on execution time in QAOA (p=1). QAOA iteratively evaluates a parameterized quantum circuit. In each iteration, the circuit is run, measured, and the expectation value is estimated. This process requires a large number of shots and repeated transpile steps. Two-qubit gate density and measurement statistics increase the time cost. The main determinant is the number of expectation calls. COBYLA is faster with fewer evaluations. L-BFGS-B provides semi-Newtonian directional information but requires more calls. SLSQP takes the longest due to the cost of numerical differentiation and scheduling. This finding confirms that the duration is essentially determined by the number of quantum evaluations. The Qiskit Optimization library developed by IBM was used to create quantum optimization models. The mathematical modeling of the path planning problem is defined in the form of a binary quadratic program containing linear constraints and a linear objective function. This model is defined on valid edges to ensure the solution set is secure. After identifying the valid edges, the resulting quadratic program model was optimized using the QAOA algorithm. At this stage, parameterization operations were performed via quantum circuits, and for the classical optimization process, the COBYLA algorithm, which is non-derivative-based and provides effective results for low-dimensional problems, was chosen. Thanks to this integrated structure, both obstacle detection and route optimization could be effectively performed using quantum computing. The parallel superposition property of quantum algorithms allows for the simultaneous evaluation of alternative paths, especially in large graph structures. This structure offers a significant advantage over the sequential search process of traditional algorithms such as Dijkstra or A* (Wang et al., 2018). Furthermore, the parameterization process carried out via quantum circuits has been supported by hardware-compatible variational optimization techniques, and the accuracy of the results has been tested with classical validators.

The image below shows the optimal solution path obtained in a graph-based path planning problem where all edges are considered valid without defining any obstacles. This scenario represents the algorithm's baseline scenario. During the solution process, the accessibility of all edges allowed the algorithm to operate on a fully connected solution space. Thus, the resulting path was determined to be the route with the lowest total cost. The experimental results of the study show that route planning processes carried out with quantum algorithms can offer more efficient and effective solutions compared to traditional methods, especially in dynamic environmental conditions and directed graphs full of obstacles. This method offers high potential not only in fixed routes but also in areas such as multi-target task assignment, mobile robotic navigation, and autonomous vehicle route planning. However, the limitations in the number of qubits and noise levels of current quantum hardware cause the algorithms to operate with limited accuracy in more complex problems. Therefore, integrating error-corrected quantum systems and noise-aware QAOA approaches in future studies can further improve the optimization quality (Das and Chakrabarti, 2005).

Conclusion

This study proposes a two-stage hybrid quantum approach involving filtering invalid edges in directed and obstacle-constrained graphs using a quantum oracle with the Grover algorithm, followed by route optimization only on the valid subgraph using the QAOA algorithm. In the literature, the Grover algorithm is frequently applied in fields such as data search, classification, or constraint satisfaction problems, but its use for systematically excluding invalid structures at the edge level in directed path planning problems is quite limited (Campbell et al., 2019). In this context, our study expands the application area of the algorithm by adapting the Grover oracle for edge-based obstacle detection. The success of the QAOA algorithm in classical combinatorial optimization areas such as Max-Cut, independent set, or task assignment problems is well-known. However, many of these applications do not directly include invalidity constraints that limit the solution space, leading the algorithm to perform computations on invalid solutions as well. In this study, thanks to pre-filtering with the Grover algorithm, QAOA is enabled to work only on physically possible paths. This approach aligns with constraint-aware QAOA models that work only on valid solutions and facilitate parameter learning. This study is one of the rare examples that applies these models to the directional path planning problem. The linear constraints used in the modeling consistently limited directional transitions between source and destination nodes, and flow balancing based on the graph topology was successfully achieved. Interpretation of the experimental outputs confirmed that the proposed method narrows the solution space by excluding invalid paths thanks to the Grover preprocessing step, and that QAOA performs optimization in a narrower but meaningful search space. This feature eliminates the recalculation requirements encountered in classical algorithms, offering practical advantages, especially for route optimization in stationary environments. In conclusion, this study proposes a hybrid quantum solution model, not yet widely known in the literature, by integrating Grover and QAOA algorithms and applying them to the path planning problem in directional and obstacle graphs. Considering existing NISQ hardware, the parametric circuit structure has been simplified and a feasible architecture has been created by supporting it with classical optimization (COBYLA). In future studies, extending the model with multi-target route planning, dynamic obstacle scenarios, and fault-tolerant QAOA variations could enable the method to make more comprehensive contributions to both the theoretical and applied quantum optimization literature.

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Conflict of interest

The authors confirm that there is no conflict of interest involve with any parties in this research study.

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